Journal of Sound and Vibration (2001) **248**(2), 215–233
doi:10.1006/jsvi.2001.3678, available online at http://www.idealibrary.com on IDEAL®

FINITE ELEMENT TRANSIENT DYNAMIC ANALYSIS OF LAMINATED STIFFENED SHELLS

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(*Received* 3 *March* 2000, *and in final form* 10 *November* 2000)

The present work describes the transient dynamic response of unstiffened/stiffened composite plates/shells using finite element method. Composite panels find wide applications in aerospace, marine and other engineering because of its high strength to weight ratios. These structures are often subjected to air-blast loading, underwater shock etc., which requires a thorough dynamic response analysis under such loading. A modified approach of shell and stiffener modelling is adopted here using an eight-noded isoparametric quadratic element for the shell and a three-noded curved stiffener element for the stiffeners on the concept of equal displacements at the shell–stiffener interface. The present formulation obviates the need for imposing the mesh line along the stiffeners; rather it accommodates the stiffeners elegantly anywhere placed arbitrarily inside the element with computational efficiency. Newmarks method for direct time integration has been adopted for the solution of the governing equation for undamped motion. The transient dynamic response of stiffened and unstiffened structures subjected to various kinds of time variant loading has been studied and the results are compared with the published ones.

1. INTRODUCTION

Stiffened plates and shells made of fibre-reinforced composite materials are considered for application in aerospace engineering and marine engineering owing to its superior strength to weight ratio, better durability, excellent damage tolerance and many other qualities. With the increased use of composite materials in various structural applications, the subject of transient response of composite unstiffened and stiffened panels has received widespread attention. These structures are often subjected to air-blast loading and underwater shock. Hence, a detailed transient dynamic response analysis is required so as to assure structural safety and durability under adverse loading conditions.

Liew *et al.* [1] recently presented a survey on vibration of shell panels which covers a large literature on the subject. Christoforou and Swanson [2] have presented an analytical solution for simply supported orthotropic cylindrical shells subjected to impulsive loading. Lin and Lee [3] have studied the dynamic response analysis of laminated shells using finite element method. Khedeir and Reddy [4] have presented an exact solution to predict dynamic response of cross-ply cylindrical shell subjected to explosive pressure. Heppler [5] has used finite element method for the elastic response of the shell roof structure under blast loaded environment. Liew and Lim [6–8] have presented p-Ritz method for predicting the

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vibratory behaviour of shallow cylindrical and doubly curved panels. The method is an analytic type wherein the entire domain is represented by admissible functions considering the boundary conditions and is basically a global approach. Jiang and Olson [9] have used the transversely curved finite strip and beam elements along the nodal line for the linear and non-linear dynamic analysis of stiffened plates and shells. Chang and Dade [10] have presented dynamic response analysis of stiffened shells using spline Gauss collocation method. Sinha and Mukhopadhyay [11] have presented a finite element analysis of stiffened plates and shells using 36 degrees of freedom, higher order arbitrary triangular shaped shallow shell finite element and Newmark's method for direct time integration for the solution of governing equation. The stiffeners can be oriented in any direction but they have to pass through two opposite edges of the element. It is not possible to model stiffeners which are required to pass through two adjacent nodal lines of the plate/shell element. The method is applicable in the analysis of thin shallow shell structures and cannot be applied to deep and thick shells. Moreover, the element cannot account for transverse shear effects, which is very much necessary in laminated composite plate/shell analysis. Houlston [12] has analyzed the plates and stiffened panels subjected to air-blast loading using finite strip method. Houlston and DesRochers [13] have presented non-linear structural response of ship panels subjected to air-blast loading using ADINA finite element package and experimental displacement time history for a stiffened panel. Gong and Lam [14] have carried out the transient response of stiffened composite submerged hull and floating ship section subjected to underwater shock. They have used doubly asymptotic approximation (DAA) method to study the fluid–structure interactions. The fluid–structure coupled equation, relating structure response to fluid impulsive loading is solved using coupled finite element and boundary element codes. The finite element model for the submersible hull was created using MSC/PATRAN Version 5.0. Moreover the literature on transient dynamic response of composite stiffened panels is scanty.

The present work deals with the transient dynamic response of composite stiffened plates/shells using finite element method and Newmark's method for direct time integration to solve the governing undamped equation of motion. Formulation of the general curved shell element using the eight-noded isoparametric quadratic element has been derived on the basis of Mindlin–Reissner's theory and satisfying C° continuity for the interpolation functions. The element is obtained by defining two radii of curvature whose ratio to the element characteristic dimension may be up to one, i.e., deep shell and by using a three-dimensional curvilinear co-ordinate system to which the shell behaviour may be referred. The element takes into account transverse shear strains and permits both thin and moderately thick shells. The stiffener has been modelled as a three-noded curved beam element having the same displacement functions as the shell. The formulation takes care of arbitrarily oriented stiffeners by adopting suitable transformations from stiffener nodes to shell nodes irrespective of its position inside the shell element. With the present formulation, the stiffener can be positioned anywhere inside the element, this aspect has been taken care by using suitable transformations. This formulation overcomes the complication of aligning the mesh line along the stiffeners, which has been a requirement for all commercial finite element packages. The same formulation is extended, in the present paper, to the transient dynamic response of composite stiffened plates/shells under various time varying loads. The results for the isotropic unstiffened and stiffened shells and composite plates obtained by the present method are compared with published ones. Few examples have been suggested and analyzed using the present formulation.

2. FINITE ELEMENT FORMULATION

2.1. SHELL ELEMENT

Based on the assumptions adopted by Mindlin and Reissener, a C° continuity large curvature shell element having eight nodes with five degrees of freedom at each node is

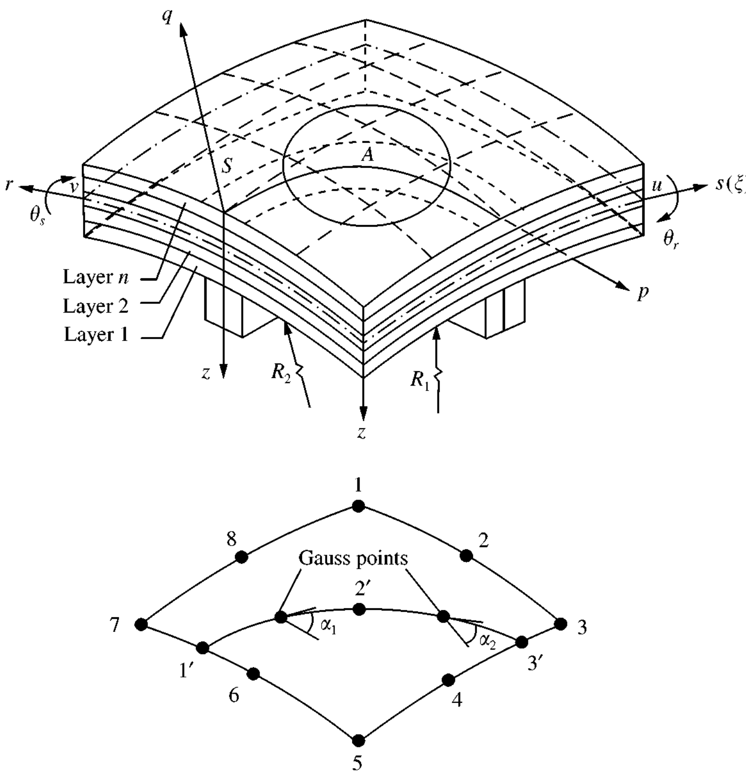


Figure 1. Orthogonal curvilinear co-ordinate system on laminated shell surface S.

developed. The effective displacement components at a node 'j' of the shell element are u_j, v_j, w_j, θ_{sj} and θ_{rj}. On the shell surface 'S' (Figure 1), two orthogonal intrinsic co-ordinates 's' and 'r', also known as curvilinear co-ordinates, can be defined from the global co-ordinate values of x, y and z as in reference [15],

$$s(\xi) = \int_0^{\xi} \sqrt{\left(\left(\frac{\mathrm{d}x}{\mathrm{d}\xi}\right)^2 + \left(\frac{\mathrm{d}y}{\mathrm{d}\xi}\right)^2 + \left(z\mathrm{d}\xi\right)^2\right)}\,\mathrm{d}\xi,$$

$$r(\eta) = \int_0^{\eta} \sqrt{\left(\left(\frac{\mathrm{d}x}{\mathrm{d}\eta}\right)^2 + \left(\frac{\mathrm{d}y}{\mathrm{d}\eta}\right)^2 + \left(\frac{\mathrm{d}z}{\mathrm{d}\eta}\right)^2\right)}\,\mathrm{d}\eta. \tag{1}$$

ξ and η are valid in the region (− 1 to 1).

The curvatures of the curvilinear co-ordinates can be evaluated by

$$\frac{1}{R_1} = \frac{1}{\left(\sqrt{\left(\frac{\partial x}{\partial \xi}\right)^2 + \left(\frac{\partial y}{\partial \xi}\right)^2 + \left(\frac{\partial z}{\partial \xi}\right)^2}\right)^3}\; C_1$$

and

$$\frac{1}{R_2} = \frac{1}{\left(\sqrt{\left(\frac{\partial x}{\partial \eta}\right)^2 + \left(\frac{\partial y}{\partial \eta}\right)^2 + \left(\frac{z}{\partial \eta}\right)^2}\right)^3}\; C_2, \tag{2}$$

where

$$C_1 = \left[\left(\frac{\partial y}{\partial \xi}\frac{\partial^2 z}{\partial \xi^2} - \frac{\partial z}{\partial \xi}\frac{\partial^2 y}{\partial \xi^2}\right)^2 + \left(\frac{\partial z}{\partial \xi}\frac{\partial^2 x}{\partial \xi^2} - \frac{\partial x}{\partial \xi}\frac{\partial^2 z}{\partial \xi^2}\right)^2 + \left(\frac{\partial x}{\partial \xi}\frac{\partial^2 y}{\partial \xi^2} - \frac{\partial y}{\partial \xi}\frac{\partial^2 x}{\partial \xi^2}\right)^2\right]^{1/2}$$

and

$$C_2 = \left[\left(\frac{\partial y}{\partial \eta}\frac{\partial^2 z}{\partial \eta^2} - \frac{\partial z}{\partial \eta}\frac{\partial^2 y}{\partial \eta^2}\right)^2 + \left(\frac{\partial z}{\partial \eta}\frac{\partial^2 x}{\partial \eta^2} - \frac{\partial x}{\partial \eta}\frac{\partial^2 z}{\partial \eta^2}\right)^2 + \left(\frac{\partial x}{\partial \eta}\frac{\partial^2 y}{\partial \eta^2} - \frac{\partial y}{\partial \eta}\frac{\partial^2 x}{\partial \eta^2}\right)^2\right]^{1/2}. \quad (3)$$

The strain–displacement equation of the shear deformable theory of doubly curved shells [16] has been considered in the present formulation for the strain values as

$$\begin{Bmatrix} \varepsilon_s \\ \varepsilon_r \\ \varepsilon_{rs} \\ \varepsilon_{sz} \\ \varepsilon_{rz} \end{Bmatrix} = \begin{Bmatrix} \varepsilon_s^0 \\ \varepsilon_r^0 \\ \varepsilon_{rs}^0 \\ \varepsilon_{sz}^0 \\ \varepsilon_{rz}^0 \end{Bmatrix} + z \begin{Bmatrix} k_s \\ k_r \\ k_{rs} \\ 0 \\ 0 \end{Bmatrix} \quad (4)$$

with

$$\begin{Bmatrix} \varepsilon_s^0 \\ \varepsilon_r^0 \\ \varepsilon_{rs}^0 \\ k_s \\ k_r \\ k_{rs} \\ \varepsilon_{sz}^0 \\ \varepsilon_{rz}^0 \end{Bmatrix} = \begin{bmatrix} \dfrac{\partial u}{\partial s} - \dfrac{w}{R_1} \\ \dfrac{\partial v}{\partial r} - \dfrac{w}{R_2} \\ \dfrac{\partial u}{\partial r} + \dfrac{\partial v}{\partial s} \\ -z\dfrac{\partial \theta_r}{\partial s} \\ -z\dfrac{\partial \theta_s}{\partial r} \\ -z\left(\dfrac{\partial \theta_s}{\partial s} + \dfrac{\partial \theta_r}{\partial r}\right) - \dfrac{1}{2}\left(\dfrac{1}{R_1} - \dfrac{1}{R_2}\right)\left(\dfrac{\partial v}{\partial s} - \dfrac{\partial u}{\partial r}\right) \\ \theta_r + \dfrac{\partial w}{\partial s} - \dfrac{v}{R_2} \\ \theta_s + \dfrac{\partial w}{\partial r} - \dfrac{u}{R_1} \end{bmatrix}, \quad (5)$$

where $\varepsilon_s^0, \varepsilon_r^0 \ldots$ correspond to midplane strains and k_s, k_r and k_{rs} correspond to curvatures. The constitutive equations are given by

$$\{F\} = [D]\{\varepsilon\}, \quad (6)$$

where

$$\{F\} = \{N_s \; N_r \; N_{sr} \; M_s \; M_r \; M_{sr} \; Q_s \; Q_r\}^{\mathrm{T}}. \quad (7)$$

The rigidity matrix for a laminated shell can be formulated using standard principles of mechanics of composite materials as

$$[D] = \begin{bmatrix} [A_{ij}] & [B_{ij}] & 0 \\ [B_{ij}] & [D_{ij}] & 0 \\ 0 & 0 & [S_{ij}] \end{bmatrix} \tag{8}$$

with the coupling parameters as

$$[A_{ij}],\ [B_{ij}],\ [D_{ij}] = \sum_{k=1}^{n} \int_{z_k}^{z_{k+1}} [Q_{ij}]^k (1, z, z^2)\,\mathrm{d}z \quad i, j = 1, 2, 6 \text{ and}$$

$$[S_{ij}] = \sum_{k=1}^{n} \int_{z_k}^{z_{k+1}} k_s\,[Q_{ij}]^k\,\mathrm{d}z \quad i, j = 4, 5 \quad k_s = 5/6 \text{ for rectangular cross sections,}$$

where $[Q_{ij}]^k$ are the material properties of the kth layer and A, B and D are the extensional, flexural–extensional coupling and flexural stiffness matrices of the laminate respectively.

The strain–displacement relation is given by

$$\{\varepsilon\} = [B]\,\{\delta\}, \tag{9}$$

where

$$\{\delta\}^{\mathrm{T}} = \{u_1 \;\; v_1 \;\; w_1 \;\; \theta_{s1} \;\; \theta_{r1} \cdots \theta_{s8} \;\; \theta_{r8}\}.$$

The stiffness matrix is expressed as

$$[K]_e = \int_{-1}^{+1} \int_{-1}^{+1} [B]^{\mathrm{T}} [D]\,[B]\,|J|\,\mathrm{d}\xi\,\mathrm{d}\eta. \tag{10}$$

The Jacobian matrix $|J|$ being rectangular, the determinant of such matrix cannot be calculated in the conventional manner, hence the approach adopted by Saetta and Vitaliani [15] is used in the present work.

2.2. MASS MATRIX OF SHELL ELEMENT

Consistent mass matrix formulation is used in the present work. The derivation of element mass matrix is performed in line with the element stiffness matrix. The displacement shape functions are considered to compute the consistent mass matrix.

The consistent mass matrix for the shell element is written as

$$[M]_e = \int_A [N]^{\mathrm{T}} [\bar{m}]\,[N]\,\mathrm{d}A = \int_{-1}^{+1} \int_{-1}^{+1} [N]^{\mathrm{T}} [\bar{m}]\,[N]\,|J|\,\mathrm{d}\xi\,\mathrm{d}\eta, \tag{11}$$

where $[\bar{m}] = \begin{bmatrix} \rho & 0 & 0 \\ 0 & \rho & 0 \\ 0 & 0 & \rho \end{bmatrix}$; ρ is the mass density of the shell material.

2.3. STIFFENER ELEMENT

Stiffener has been modelled as a three-noded curved line element having 5 degrees of freedom per node as in the case of shell element nodes (Figure 1). The stiffener has been modelled as a skew beam in line with the Martini and Vitaliani [17]. The derivation of stiffness matrix for the stiffener is presented briefly.

The curvilinear co-ordinate 'p' on a skew line can be defined as

$$p(\xi) = \int_{-1}^{+1} |J_{st}|\, \mathrm{d}\xi, \tag{12}$$

where

$$|J_{st}| = \left(\left(\sum_{i=1}^{3} \frac{\mathrm{d}N_i}{\mathrm{d}\xi} x_i\right)^2 + \left(\sum_{i=1}^{3} \frac{\mathrm{d}N_i}{\mathrm{d}\xi} y_i\right)^2 + \left(\sum_{i=1}^{3} \frac{\mathrm{d}N_i}{\mathrm{d}\xi} z_i\right)^2\right)^{1/2} \tag{13}$$

with

$$x = \sum_{i=1}^{3} x_i N_i, \quad y = \sum_{i=1}^{3} y_i N_i, \quad z = \sum_{i=1}^{3} z_i N_i. \tag{14}$$

The shape functions of stiffener element at the node i is expressed as

$$N_1 = \frac{-\xi(1-\xi)}{2}, \quad N_2 = (1-\xi^2), \quad N_3 = \frac{\xi(1+\xi)}{2}. \tag{15}$$

Curvature of the stiffener at any point on the beam axis can be expressed as

$$\frac{1}{R} = \frac{C}{|J_{st}|^3}, \tag{16}$$

with

$$C = \left[\left(\frac{\partial y}{\partial \xi}\frac{\partial^2 z}{\partial \xi^2} - \frac{\partial z}{\partial \xi}\frac{\partial^2 y}{\partial \xi^2}\right)^2 + \left(\frac{\partial z}{\partial \xi}\frac{\partial^2 x}{\partial \xi^2} - \frac{\partial x}{\partial \xi}\frac{\partial^2 z}{\partial \xi^2}\right)^2 + \left(\frac{\partial x}{\partial \xi}\frac{\partial^2 y}{\partial \xi^2} - \frac{\partial y}{\partial \xi}\frac{\partial^2 x}{\partial \xi^2}\right)^2\right]^{1/2}. \tag{17}$$

The displacement field of the stiffener element is given by

$$\{u'\ v'\ w'\ \theta'_p\ \theta'_q\}^{\mathrm{T}} = \sum_{i=1}^{3} N_i \{u'_i\ v'_i\ w'_i\ \theta'_{pi}\ \theta'_{qi}\}^{\mathrm{T}}. \tag{18}$$

The stiffener displacements in terms of the shell displacements, considering the stiffener inclination (α) with respect to s-axis are expressed as

$$\begin{aligned} u' &= u\cos\alpha + v\sin\alpha, \\ v' &= -u\sin\alpha + v\cos\alpha, \\ w' &= w, \\ \theta'_p &= \theta_s\cos\alpha + \theta_r\sin\alpha, \\ \theta'_q &= -\theta_s\sin\alpha + \theta_r\cos\alpha. \end{aligned} \tag{19}$$

Expressing in matrix notation,

$$\sum_{i=1}^{3} \{\delta_{st}\}_t = [\Lambda] \sum_{i=1}^{3} \{\delta\}_i \tag{20}$$

where

$$[\Lambda] = \begin{bmatrix} c & s & 0 & 0 & 0 \\ -s & c & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & c & s \\ 0 & 0 & 0 & -s & c \end{bmatrix} \quad \text{with } c = \cos\alpha \text{ and } s = \sin\alpha. \tag{21}$$

In the evaluation of $[\Lambda]$, the angle α is taken at the Gauss points considering the tangents of the stiffener element.

The generalized strain components of the stiffener with respect to the reference axis system of the stiffener is given by

$$\{\varepsilon_{st}\}^{\mathrm{T}} = \left\{ \left(\frac{\partial u'}{\partial p} + \frac{w}{R}\right) - \frac{\partial \theta'_p}{\partial p} \left(\frac{\partial w'}{\partial p} - \theta'_p - \frac{u'}{R}\right) \frac{\partial \theta'_q}{\partial p} \right\}. \tag{22}$$

Considering the eccentricity (e) of the stiffener and employing the necessary transformation that is required from one cylindrical surface to another, the strain–displacement matrix can be expressed as

$$\{\varepsilon_{st}\} = \sum_{i=1}^{3} [B_{st}]_i \ \{\delta_{st}\}_i, \tag{23}$$

where

$$[B_{st}]_i = \begin{bmatrix} \left(1 - \frac{e}{R}\right)\frac{\partial N_i}{\partial p} & 0 & \left(-\frac{N_i}{R} - e\frac{\partial^2 N_i}{\partial p^2}\right) & 0 & 0 \\ 0 & 0 & 0 & -\left(\frac{1}{(1-e/R)\,\partial N_i/\partial p}\right) & 0 \\ \left\{-\frac{1}{R}\left(1 - \frac{e}{R}\right)\right\} N_i & 0 & \left(\frac{e}{R} + \frac{1}{(1-e/R)}\right)\frac{\partial N_i}{\partial p} & -N_i & 0 \\ 0 & 0 & 0 & 0 & -\frac{1}{(1-e/R)}\frac{\partial N_i}{\partial p} \end{bmatrix}. \tag{24}$$

Since the nodes of the stiffener element are within the shell element, the nodal displacements of the stiffener element in terms of those of the shell element can be expressed as

$$\underset{(15\times 1)}{\sum_{i=1}^{3} \{\delta_{st}\}_i} = \underset{(15\times 15)}{[\Lambda]} \quad \underset{(15\times 40)}{[T]} \quad \underset{(40\times 1)}{\sum_{j=1}^{8} \{\delta\}_j}, \tag{25}$$

where $[T]$ is the matrix of transformation of displacements from stiffener nodes to shell element nodes and can be expressed as

$$[T] = \sum_{i=1}^{3} \sum_{j=1}^{8} \begin{bmatrix} N_{ij} & & & & \\ & N_{ij} & & O & \\ & & N_{ij} & & \\ & O & & N_{ij} & \\ & & & & N_{ij} \end{bmatrix}. \tag{26}$$

The stress–strain relationship for the stiffener element is given by

$$\{\sigma_{st}\} = [D_{st}]\{\varepsilon_{st}\}, \tag{27}$$

where $[D_{st}]$ is the rigidity matrix of the stiffener.

The stiffness matrix of the stiffener is evaluated considering the strain energy of the stiffener element and is expressed as

$$[K_{st}] = \int_p [T]^{\mathrm{T}}[\Lambda]^{\mathrm{T}}[[B_{st}]^{\mathrm{T}}[D_{st}][B_{st}]][\Lambda][T]\,\mathrm{d}p. \tag{28}$$

In a stiffener having straight configuration, the above equation can be further simplified as

$$[K_{st}] = [T]^{\mathrm{T}}[\Lambda]^{\mathrm{T}}[\bar{K}_{st}][\Lambda][T], \tag{29}$$

where $[\bar{K}_{st}]$ is the elastic stiffness matrix of the stiffener element irrespective of the orientation of the stiffener within the shell element. Since $[\Lambda]$ and $[T]$ are independent of the co-ordinate system, they can be taken out of the integration.

Therefore,

$$[\bar{K}_{st}] = \int_p [B_{st}]^{\mathrm{T}}[D_{st}][B_{st}]\,\mathrm{d}p. \tag{30}$$

Since, $[B_{st}]$ is a function of 'p' only, the expression for $[\bar{K}_{st}]$ can be written as

$$[\bar{K}_{st}] = \int_{-1}^{+1} [B_{st}]^{\mathrm{T}}[D_{st}][B_{st}]|J_{st}|\,\mathrm{d}\xi. \tag{31}$$

For a curved stiffener, the expression for stiffness matrix can be expressed for two points integration scheme as

$$[K_{st}] = [T]^{\mathrm{T}}\int_p ([\Lambda_1]^{\mathrm{T}}[\bar{K}_{st1}][\Lambda_1] + [\Lambda_2]^{\mathrm{T}}[\bar{K}_{st2}][\Lambda_2])[T]. \tag{32}$$

Here, the transformation matrix $[T]$ can only be taken out of the integration and the matrix $[\Lambda]$ is evaluated at the Gauss points of the stiffener considering the tangent angle (α) of the curved stiffener. $[K_{st1}]$ and $[K_{st2}]$ are the products $[B]^{\mathrm{T}}[D][B]$ evaluated at the Gauss points 1 and 2 respectively for the stiffness matrix of the curved stiffener.

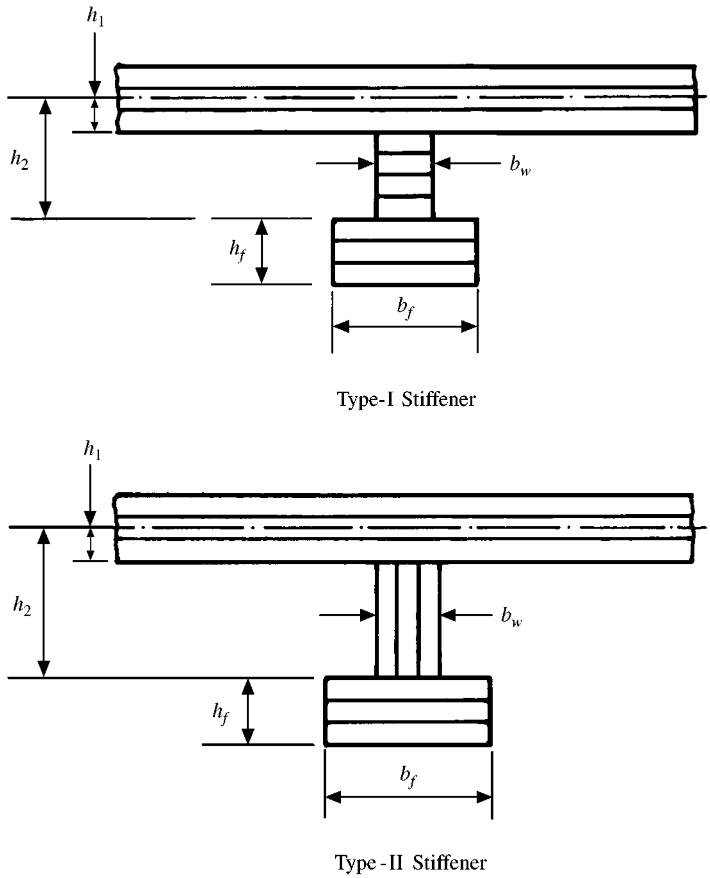


Figure 2. Types of T-stiffener (a) Type-I stiffener and (b) Type-II stiffener.

2.4. RIGIDITY MATRIX OF STIFFENER ELEMENT

The rigidity matrix due to type-I stiffener (Figure 2) can be expressed as

$$[D_{st}] = \begin{bmatrix} A_{11w}h + A_{22f}b_f & B_{22f} + A_{11w}he & A_{16w}h & -B_{16w}h + B_{26f}b_f \\ B_{22f}b_f + A_{11w}he & D_{22f}b_f + A_{11w}h_c & A_{16w}he & D_{26f}b_f - B_{16w}he \\ A_{16w}h & A_{16w}he & k_s A_{66w}h & -B_{66w}h \\ B_{16f}b_f - B_{26w}h & D_{26f}b_f - B_{16w}he & -B_{66w}h & \frac{1}{6}[Q_w h b_w^3 + Q_f b_f h_f^3] \end{bmatrix}, \tag{33}$$

where

$$h_c = \frac{h^3}{12} + he^2, \quad Q_W = Q_{66w} + Q_{55w}, \; Q_f = Q_{66f} + Q_{44f}.$$

Similarly, the rigidity matrix of type II stiffener (Figure 2) can be derived as

$$[D_{st}] = \begin{bmatrix} A_{22w}b_w + A_{22f}b_f & B_{22w} + B_{22f}b_f & 0 & B_{26w}b_w + B_{26f}b_f \\ B_{22w}b_w + B_{22f}b_w & D_{22w}b_w + D_{22f}b_f & 0 & D_{26w}b_w + D_{26f}b_f \\ 0 & 0 & k_s A_{55w}b & 0 \\ B_{26w}b_w + B_{26f}b_f & D_{26w}b_w + D_{26f}b_f & 0 & \frac{1}{6}[Q_{w1}hb_w^3 + Q_{f1}b_f h_f^3] \end{bmatrix}, \quad (34)$$

where $Q_{w1} = Q_{66w} + Q_{55w}$, $Q_{f1} = Q_{66f} + Q_{55f}$ and k_s is the shear correction factor, considered as 5/6 for rectangular cross-sections. The subscripts w and f with the terms in $[D_{st}]$ correspond to the web and flange of the T-stiffener respectively.

A, B and D as the extensional, flexural–extensional coupling and flexural stiffness matrices of the laminate, respectively, can be expressed as

$$[A_{ij}], [B_{ij}], [D_{ij}] = \sum_{k=1}^{n} \int_{z_k}^{z_{k+1}} [Q_{ij}]^k (1, z, z^2)\, \mathrm{d}z, \quad i, j = 1, 2, 6.$$

2.5. MASS MATRIX OF STIFFENER

The consistent mass matrix of the stiffener element is derived on the same line as for the elastic stiffness matrix

$$[M_{st}] = [T]^{\mathrm{T}} [\Lambda]^{\mathrm{T}} [\bar{M}_{st}] [\Lambda] [T], \quad (35)$$

where

$$[\bar{M}_{st}] = \int_p [N_i]^{\mathrm{T}} [m_{st}] [N_i]\, \mathrm{d}p \quad \text{with } [m_{st}] = \rho_{st} A_{st}.$$

ρ_{st} is the density of the stiffener material and A_{st} is the area of the stiffener.

For the curved stiffener element the evaluation of mass matrix $[M_{st}]$ will follow steps similar to that of the stiffness matrix in equation (32).

2.6. GOVERNING EQUATION

The equilibrium equation for an undamped stiffened structural system for a load having an arbitrary time variation can be expressed as

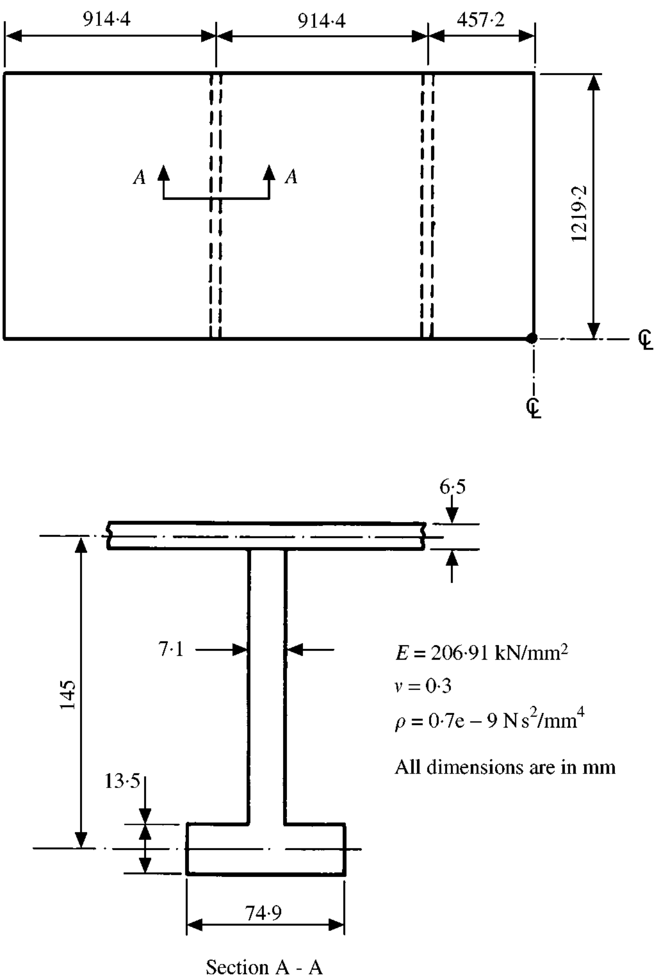
$$[K]\{\delta\} + [M]\{\ddot{\delta}\} = \{F(t)\}, \quad (36)$$

where $\{F(t)\}$ is the overall load vector, which is dependent on both space and time, $[K]$ and $[M]$ are the assembled stiffness and consistent mass matrices, $\{\delta\}$ and $\{\ddot{\delta}\}$ are the nodal displacement and acceleration vectors. Explicit time integration technique is adopted following the Newmark method [18, 19] for the solution of the resulting global set of equations. To obtain integration accuracy and stability, the parameters β and α required in Newmark method are taken as 0·5 and 0·25 respectively.

3. NUMERICAL EXAMPLES

3.1. EXAMPLE-1: TRANSIENT DYNAMIC RESPONSE OF A 5-BAY DRES STIFFENED PANEL SUBJECTED TO AIR-BLAST LOADING

A rectangular clamped stiffened plate, with four identical T-stiffeners in one direction only (Figure 3), is subjected to blast loads in a blast chamber. The variation of average pressure, due to blast load with time is shown in Figure 4. The large deflection elasto-plastic response of the structure under the MINOR SCALE blast load has been investigated by Houlston [12] using finite strip method. Sinha [20] used a high precision triangular stiffened shallow shell element to study the response of the DRES stiffened panel. Figure 5 shows the finite element mesh in the quarter panel along with the stiffener location and orientation. The linear response analysis is carried out with one quarter of the plate using a time step of 0·05 msec. The displacement response at the centre of the plate is compared with that of Sinha [20] in Figure 6. The comparison shows good agreement between the two results, with the fact that the analysis carried out by the present formulation involves less degree of freedom, which is an improvement.



$E = 206{\cdot}91$ kN/mm^2

$\nu = 0{\cdot}3$

$\rho = 0{\cdot}7\mathrm{e} - 9$ N s^2/mm^4

All dimensions are in mm

Figure 3. Five-bay clamped DRES stiffened panel.

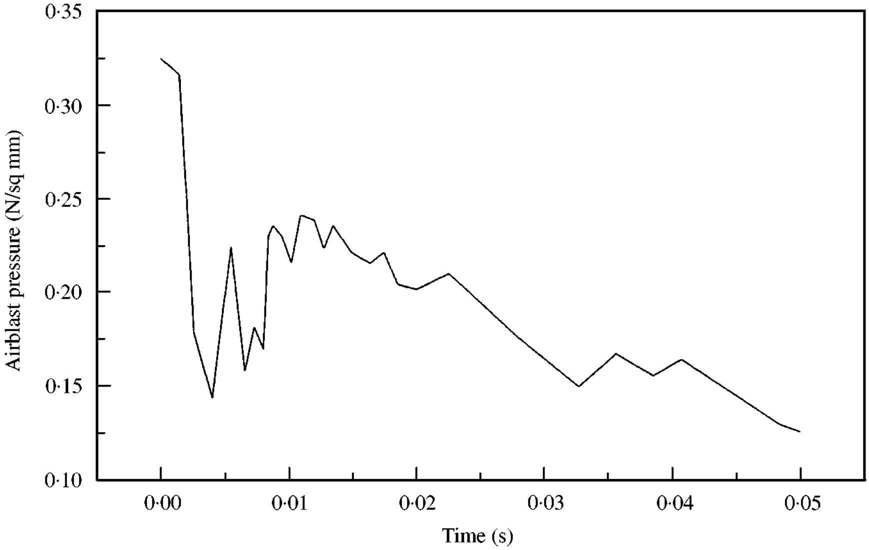


Figure 4. Air-blast loading history for 5-bay DRES stiffened panel.

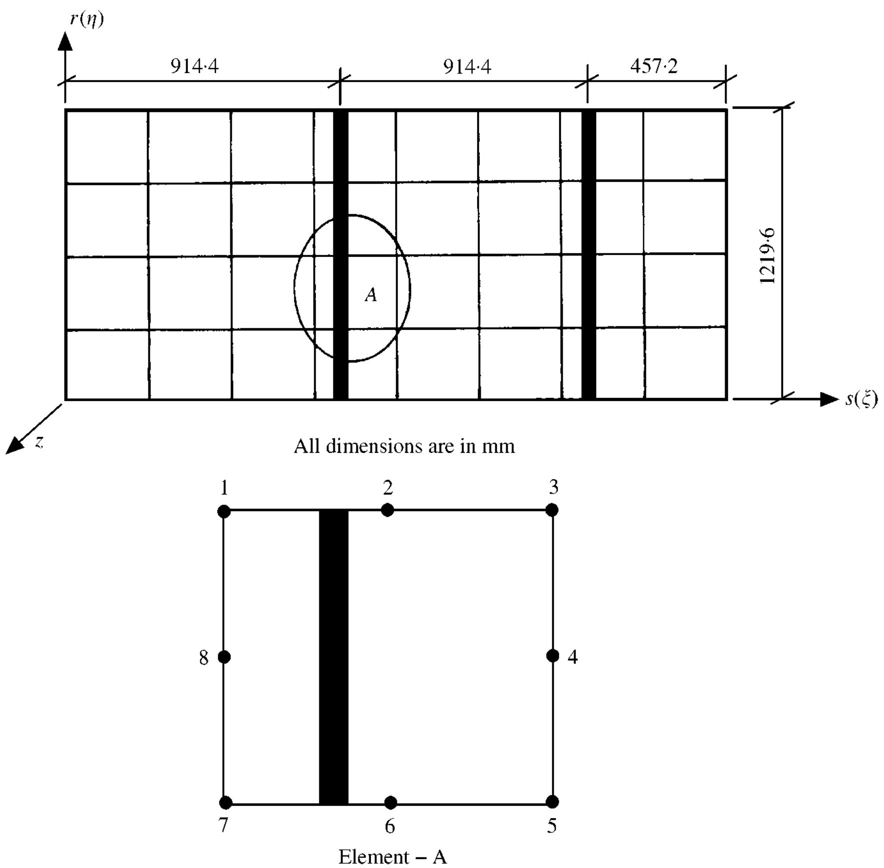


Figure 5. Finite element mesh showing the location and orientation of stiffener in the 5-bay DRES stiffened panel.

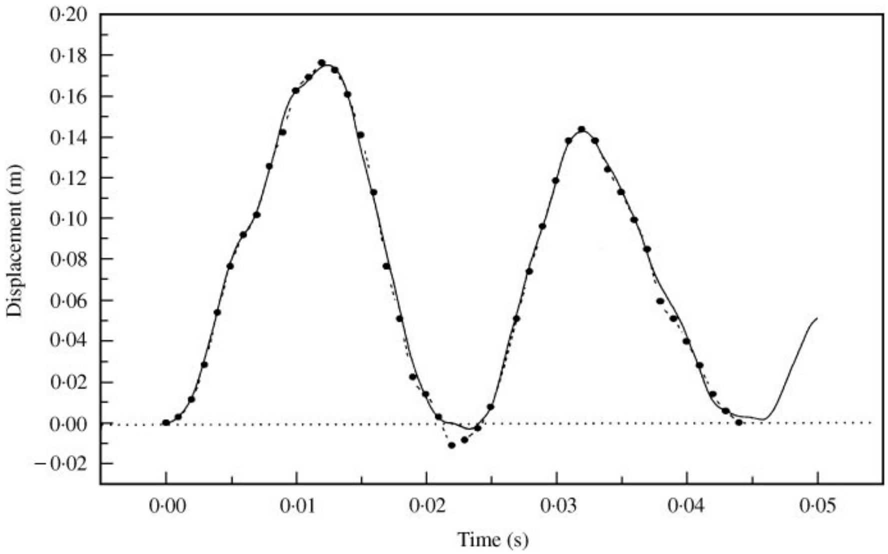


Figure 6. Vertical displacement response at the centre of 5-bay clamped DRES stiffened plate panel. ——, present; - - -●- - -, Sinha [20].

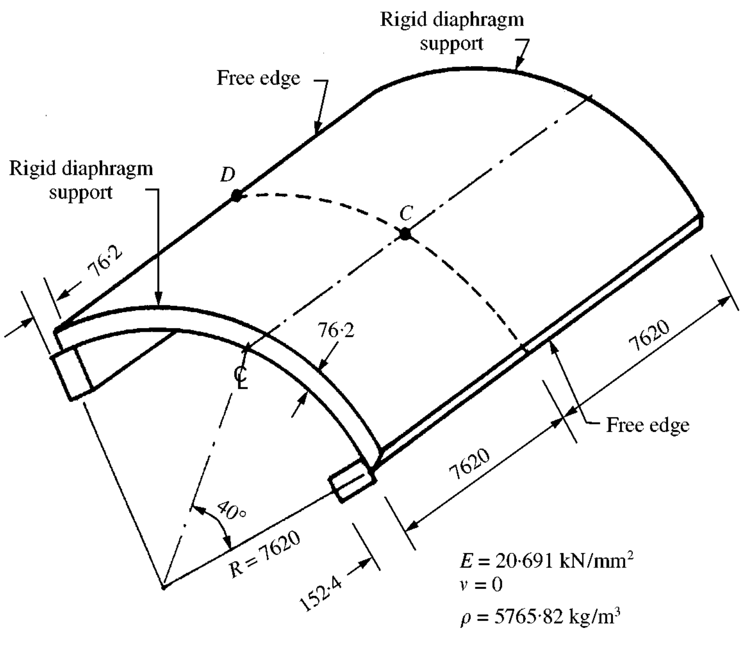


Figure 7. Simply supported stiffened shell-roof structure.

3.2. EXAMPLE-2: TRANSIENT DYNAMIC RESPONSE OF UNSTIFFENED/STIFFENED CYLINDRICAL SHELL UNDER SELF-WEIGHT

Transient dynamic response of a cylindrical shell with two straight edge stiffeners and supported on rigid diaphragms at the curved edges (Figure 7) subjected to uniformly

TABLE 1

Free Vibration analysis of stiffened cylindrical shell (Figure 7)

Mode	Symmetric condition	Present	Jiang and Olson [6]	Sinha [20]
1	Anti symmetric	1·818	1·845	1·812
2	Symmetric	2·020	1·977	1·938
3	Anti symmetric	4·330	4·259	4·224
4	Symmetric	6·760	7·042	6·742

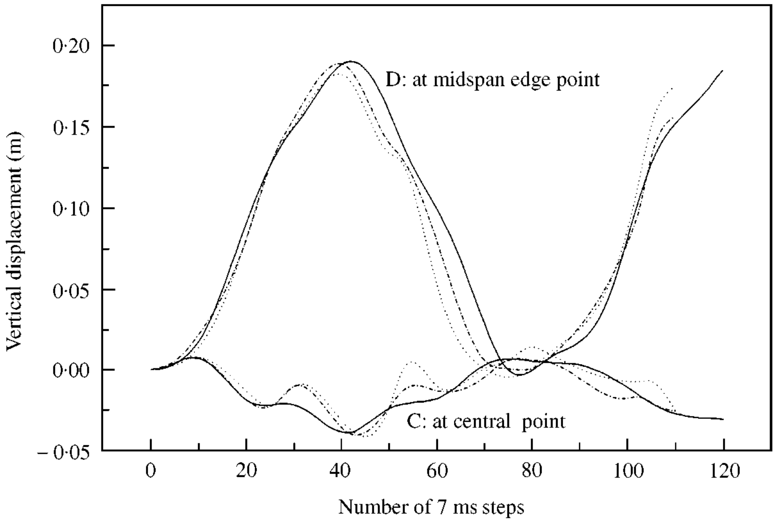


Figure 8. Linear elastic transient response of simply supported shell-roof structure under a dead weight step load. ——, Present; -·-·-·-, Heppler [5]; ········, Sinha [20].

distributed step load has been considered for analysis. The geometrical and material properties of the shell and stiffener are also described in Figure 7. The same problem has been solved by Heppler [5] using finite element method and Jiang and Olson [9] for the comparison of the large-deflection elasto-plastic response between stiffened and unstiffened isotropic shell panels. Sinha and Mukhopadhyay [11] investigated the same problem using higher order triangular shallow shell element. Table 1, shows a comparison of the natural frequencies for the first few modes for the stiffened shell-roof using the present formulation with that of the other researchers.

Transient response analysis is carried out with self-weight of the shell acting as a step load and a time step of 7 ms has been considered for the analysis. Figure 8 shows the comparison of the vertical displacement response values of Happler [5] and Sinha [20] for the unstiffened cylindrical shell at central point and at mid-span edge point with the present formulation. The displacement response of stiffened cylindrical shell (Figure 7) has also been compared with Sinha [20] in Figure 9 and is found to be in good agreement.

3.3. EXAMPLE-3: TRANSIENT DYNAMIC RESPONSE OF UNSTIFFENED AND STIFFENED CROSS-PLY LAMINATED PLATE UNDER STEP LOAD

The transient response analysis of a rectangular unstiffened cross-ply laminate $[0/90]_T$ is carried out using the present formulation. The laminate is simply supported along all the

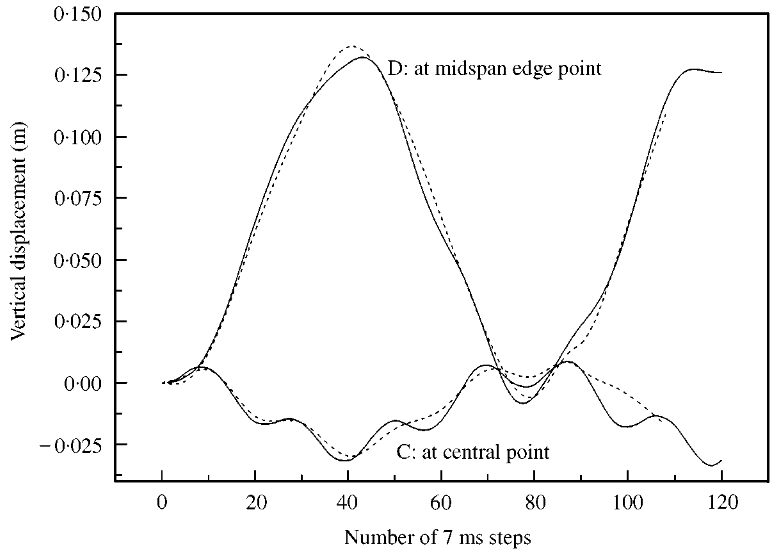


Figure 9. Transient dynamic response of stiffened cylindrical shell-roof. ——, Present; - - - - - -, Sinha [20].

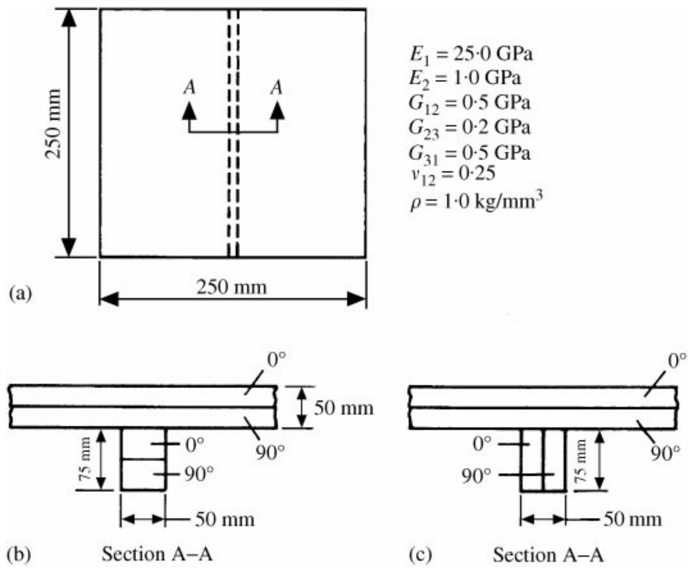


Figure 10. Simply supported laminated stiffened plate with a central eccentric stiffener: (b) stiffener with parallel laminations, Type-A, and (c) stiffener with perpendicular laminations, Type-B.

edges (Figure 10) and is subjected at time $t = 0$ to a uniform pressure (step load) of magnitude 100 kPa. The geometry and material properties are shown in Figure 10. The transient dynamic response analysis of the unstiffened laminate has been carried out earlier by Meimaris and Day [21] using 20-noded parabolic isoparametric solid elements. The response is studied at the centre of the plate with a time step of 1 μs. The response of the central displacement of unstiffened laminated plate using present formulation and that of Meimaris and Day [21] is presented in Figure 11. The plate being a thick one with thickness

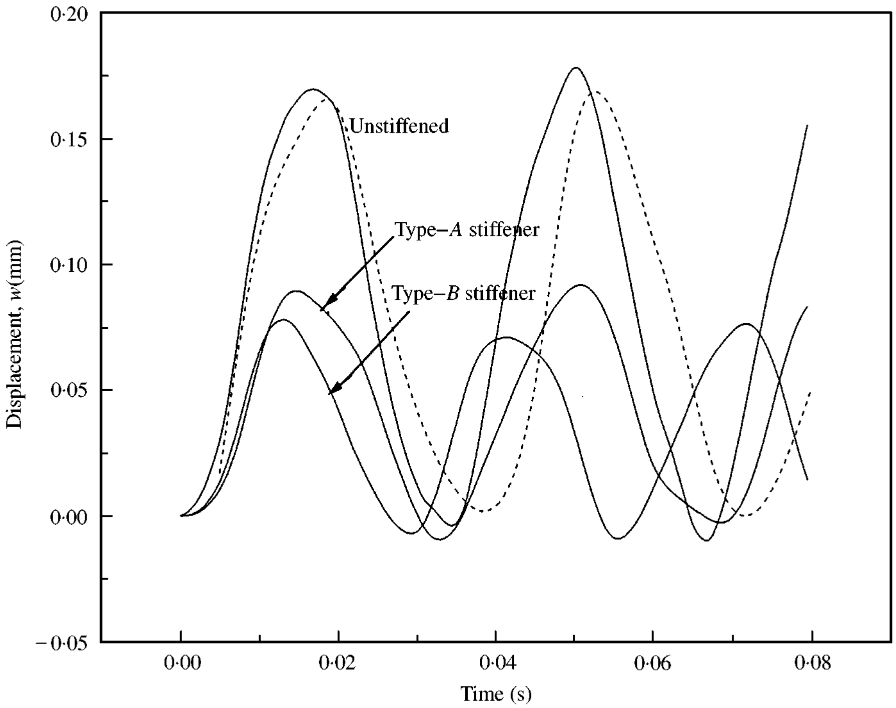


Figure 11. Displacement response at the centre of laminated stiffened and un-stiffened plate: ——, Present; ------, Meimaris [21].

ratio 5, the effects of transverse shear are considerable. The close agreement between the two results indicates the capability of analysing thick plates by the present element and is comparable to a 20-noded isoparametric solid element.

To establish the capability of analyzing composite stiffened laminate, an additional example is suggested in the present investigation. The plate is stiffened with a central rectangular stiffener having parallel lamination (Type-A) and perpendicular lamination (Type-B) with $[0/90]_T$ laminates (Figure 10). Displacement response is studied at the centre of the plate with a time step of 1 μs. The response of the central displacement of stiffened laminates for both the types of stiffeners (Type-A and B) using present formulation is presented in Figure 11. Significant reduction of displacement is noticed with the introduction of stiffeners.

3.4. EXAMPLE-4: TRANSIENT DYNAMIC RESPONSE ANALYSIS OF UNSTIFFENED AND STIFFENED CROSS-PLY LAMINATED SPHERICAL SHELL PANEL SUBJECTED TO EXTERNAL PRESSURE LOAD

The transient response analysis of a simply supported spherical shell under external pressure load has been attempted by Reddy and Chandrasekhara [22] for the geometrically non-linear transient analysis of laminated spherical shells. The geometric parameters and material properties used are indicated in Figure 12.

In the present investigation a 4×4 mesh for the quarter shell has been considered for the displacement response at the centre of the shell. The results are compared with the solution of Reddy and Chandrasekhara [22] in Figure 13. The comparison of the results for the unstiffened shell shows good agreement.

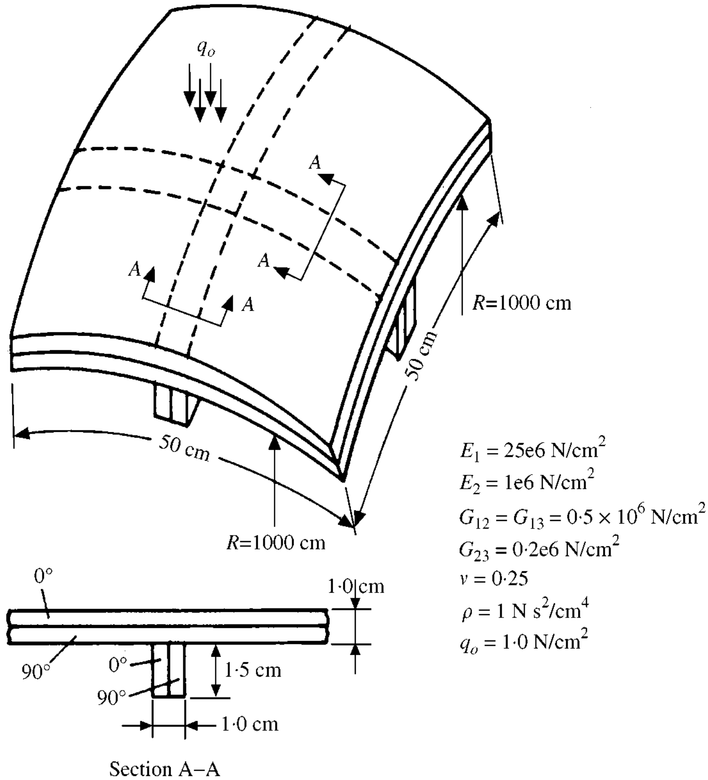


Figure 12. Simply supported laminated stiffened spherical shell under uniformly distributed load.

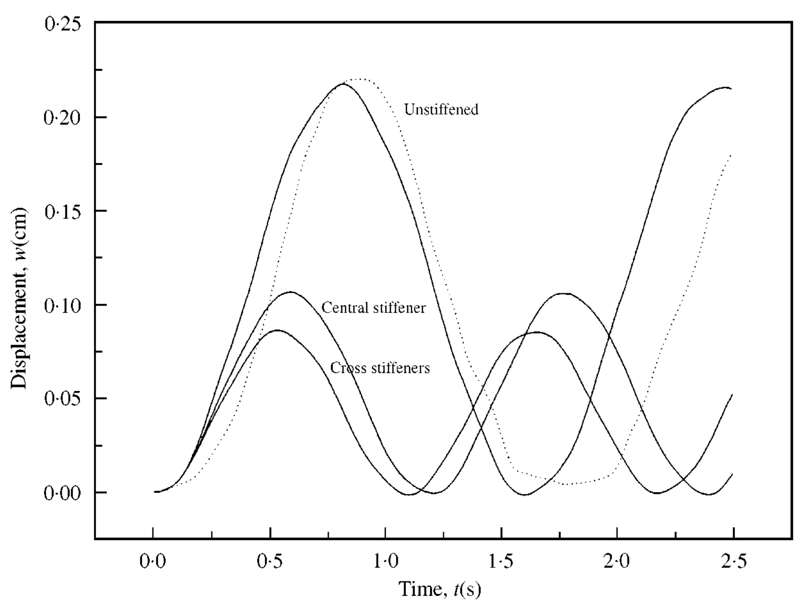


Figure 13. Displacement response at the centre of laminated unstiffened and stiffened spherical shell panel: ——, Present; ·······, Reddy [22].

In the next part of the investigation, as an additional example, the same shell panel is stiffened with stiffeners having lamination perpendicular to shell surface. The stiffener of dimension 1 cm (breadth) × 1·5 cm (height) is placed centrally and subsequently as cross stiffeners of same dimensions for linear transient dynamic analysis. Figure 13 shows the variation in displacements with respect to time (t) for the stiffened panels with central and cross stiffeners separately, in addition to the unstiffened panel. Significant decrease in the displacement values is noticed with the introduction of stiffeners to the laminated panel.

4. CONCLUSION

Transient dynamic response analysis of composite stiffened plates and shells has been carried out in the present investigation using an 8-noded curved quadratic isoparameter element for the shell and a three-noded curved beam element for the stiffener. A modified formulation for the shell and stiffener is presented. Stiffened panel is modelled based on compatibility of displacements at the shell–stiffener interface. The present formulation takes care of any number of arbitrarily oriented stiffeners inside the shell element with more computational efficiency. Varieties of structural panels with possible arbitrary loading cases are considered here for the validation of the present formulation. Results obtained using the present formulation are in good agreement with those published in literature. To establish the efficiency of the present formulation some new examples have been included in the present investigation.

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APPENDIX A: NOMENCLATURE

A_{ij}, B_{ij}, D_{ij}	coupling parameters
M_s, M_r, M_{sr}	moments
N_s, N_r, N_{sr}	in-plane forces
Q_s, Q_r	shear forces
e	eccentricity of stiffener
h	height of stiffener
s, r and z	curvilinear co-ordinate system for the shell
p, q and z	curvilinear co-ordinate system for the stiffener
k_s	shear correction factor
u, v, w, θ_s, θ_r	displacements of shell element
u', v', w', θ'_p, θ'_q	displacements of stiffener element
$[B]$	strain–displacement matrix (derivatives of shape functions)
$[D]$	rigidity matrix of shell
$[D_{st}]$	rigidity matrix of stiffener
$[N]$	shape function matrix
$\{\delta_{st}\}_i$	displacements at the ith node of stiffener element
$\{\delta_r\}$	displacements at the rth node of shell element
$\{\sigma_{st}\}$	stresses in stiffener
$\{\varepsilon_{st}\}$	strains in stiffener
$[Q_{ij}]^k$	material properties at the kth layer